Uncertainty Analysis

Uncertainty analysis is a technique for determining an estimate for the interval about a reported result within which the true result is expected to lie with a certain degree of confidence. The technique uses the general formula:

$$U = \sqrt{\Sigma\left[\frac{U_{vi} \bullet d_{vi}}{f(V)}\right]^2}$$

where f(V) is a function of n variables in the set V, d_{vi} is the derivative of f with respect to the ith variable in the set, and U_{vi} is the uncertainty (tolerance) in the input for the ith variable. U is frequently reported as a percentage instead of a decimal fraction, so a factor of 100 can be included.

Two examples are presented, using TK Solver to perform the calculations. The first example is a simple one involving estimation of a gas density given a temperature and pressure. The second estimates the stress on a hollow shaft under torsional loading, given inputs for the OD, ID, and torque.

Example 1: Gas Density

Here are the TK Solver rules used to solve for the density of a gas:

Rule
R = 8314/M ; 8314 is the Universal Gas Constant, joule/(kmol*K)
ρ = p/(R*T)

Units are very important in this example. The pressure is assumed to be measured in units of kgf/m^2 and the temperature in degrees K. The resulting density value is in units of kgf*s^2/m/m^3. These values can be converted to any display units we desire. Here is the resulting TK Solver Variable Sheet with sample inputs:

St	Input	Name	Output	Unit	Comment
	28.97	M		kg/kmol	Molecular weight
	298	T		K	Temperature
	100	p		atm	Pressure
		R	286.987	m^2/(s^2*K)	Gas constant
		ρ	118.478	kg/m^3	Density

Next, a rule is added to compute the uncertainty in the density result. The built-in function Deriv is used to compute the required derivatives.

Rule
U = sqrt(((Deriv("p/(R*T),p",p)/ρ)*Up)^2 + ((Deriv("p/(R*T),T",T)/ρ)*UT)^2)*100

The uncertainty inputs for p and T are added and the value of U is computed. Note that Up has calculation units of kgf/m^2 just like p.

St	Input	Name	Output	Unit	Comment
	1	UT		K	Temperature uncertainty
	1	Up		atm	Pressure uncertainty
		U	1.055	%	Density uncertainty percentage

The model can also be backsolved, if necessary. Assume that an upper limit of 0.5% is placed on U with the suggestion that a tighter tolerance be placed on measuring the pressure. The following TK Solver Variable Sheet summarizes the results.

St	Input	Name	Output	Unit	Comment
					Uncertainty Calculation Example:
					Density as a function of T and p
	28.97	M		kg/kmol	Molecular weight
	298	T		K	Temperature
	100	p		atm	Pressure
		R	286.987	m^2/(s^2*K)	Gas constant
		ρ	118.478	kg/m^3	Density
	1	UT		K	Temperature uncertainty
		Up	0.370665	atm	Pressure uncertainty
	.5	U		%	Density uncertainty percentage

Example 2: Shaft Torsion

Here is the TK Solver rule used to solve for the stress on a hollow shaft under torsional load:

Rule
Txy = torque*OD/(2*pi()*(OD^4-ID^4)/32)

Here is the resulting TK Solver Variable Sheet with sample inputs:

St	Input	Name	Output	Unit	Comment
	60000	torque		lbf*in	torsional moment
	2	OD		in	outside diameter
	1	ID		in	inside diameter
		Txy	40743.7	lbf/in^2	torsional shear stress

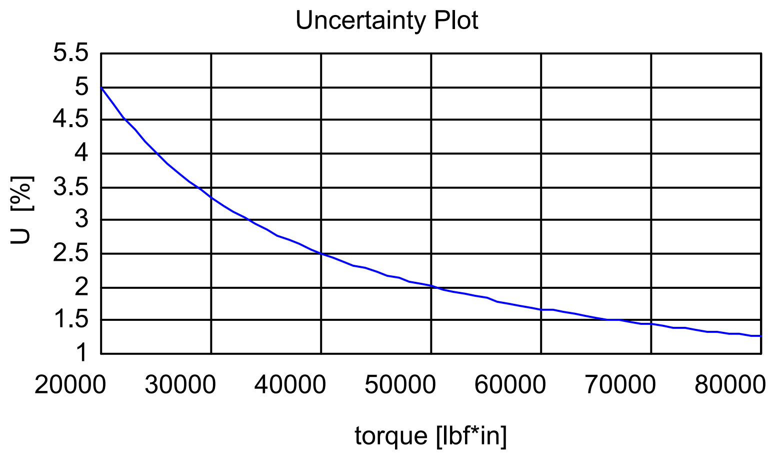
We estimate the uncertainty in the resulting Txy value with the following rules:

Rule
u1 = UOD*deriv("torque*OD/(2*pi()*(OD^4-ID^4)/32),OD",OD)/Txy
u2 = UID*deriv("torque*OD/(2*pi()*(OD^4-ID^4)/32),ID",ID)/Txy
u3 = Utorque*deriv("torque*OD/(2*pi()*(OD^4-ID^4)/32),torque",torque)/Txy
U = sqrt(u1^2 + u2^2 + u3^2)*100

The Variable Sheet summarizes the results:

St	Input	Name	Output	Unit	Comment
	60000	torque		lbf*in	torsional moment
	2	OD		in	outside diameter
	1	ID		in	inside diameter
		Txy	40743.7	lbf/in^2	torsional shear stress
	1000	Utorque		lbf*in	Uncertainty in torque
	.001	UOD		in	Uncertainty in OD
	.001	UID		in	Uncertainty in ID
		U	1.675	%	Percent uncertainty in computing Txy
		u1	-1.633E-3		OD uncertainty component
		u2	2.667E-4		ID uncertainty component
		u3	1.667E-2		torque uncertainty component

Here is a plot showing the relationship between the uncertainty in the stress, U, and the applied torque as torque varies from 20000 lbf*in to 80000 lbf*in and the shaft size remains constant.



Here is a similar plot showing the relationship between the uncertainty in the stress and the OD as the OD varies from 0.001 to 0.01 in and a constant torque of 60000 lbf*in.