Sample 2: Advanced Math -- Uncertainty Analysis

Uncertainty analysis is a technique for determining an estimate for the interval about a reported result within which the true result is expected to lie with a certain degree of confidence. The technique uses the general formula:

$$U = \sqrt{\Sigma\left[\frac{U_{vi} \cdot d_{vi}}{f(V)}\right]^2}$$

where f(V) is a function of n variables in the set V, d_{vi} is the derivative of f with respect to the ith variable in the set, and U_{vi} is the uncertainty (tolerance) in the input for the ith variable. U is frequently reported as a percentage instead of a decimal fraction, so a factor of 100 can be included.

Two examples are presented, using TK Solver to perform the calculations. The first example is a simple one involving estimation of a gas density given a temperature and pressure. The second estimates the stress on a hollow shaft under torsional loading, given inputs for the OD, ID, and torque.

Example 1: Gas Density

- Load the density.tkw file from the MathLook for Excel folder.

Here are the TK Solver rules used to solve for the density of a gas:

Rule
R = 8314/M ; 8314 is the Universal Gas Constant, joule/(kmol*K)
ρ = p/(R*T)

Units are very important in this example. The pressure is assumed to be measured in units of kgf/m^2 and the temperature in degrees K. The resulting density value is in units of kgf*s^2/m/m^3. These values can be converted to any display units we desire. Here is the resulting TK Solver Variable Sheet with sample inputs:

St	Input	Name	Output	Unit	Comment
	28.97	M		kg/kmol	Molecular weight
	298	T		K	Temperature
	100	p		atm	Pressure
		R	286.987	m^2/(s^2*K)	Gas constant
		ρ	118.478	kg/m^3	Density

- Open the subsheet for the density variable and observe the units definitions.

Output Value:	118.478
Format:	d3
Display Unit:	kg/m^3
Calculation Unit:	kgf*s^2/m/m^3
Comment:	Density

Next, a rule was added to compute the uncertainty in the density result. The built-in function Deriv is used to compute the required derivatives.

Rule
U = sqrt(((Deriv("p/(R*T),p",p)/ρ)*Up)^2 + ((Deriv("p/(R*T),T",T)/ρ)*UT)^2)*100

The uncertainty inputs for p and T are added and the value of U is computed. Note that Up has calculation units of kgf/m^2 just like p.

St	Input	Name	Output	Unit	Comment
	1	UT		K	Temperature uncertainty
	1	Up		atm	Pressure uncertainty
		U	1.055	%	Density uncertainty percentage

The model can also be backsolved, if necessary. Assume that an upper limit of 0.5% is placed on U with the suggestion that a tighter tolerance be placed on measuring the pressure. The following TK Solver Variable Sheet summarizes the results.

St	Input	Name	Output	Unit	Comment
					Uncertainty Calculation Example:
					Density as a function of T and p
	28.97	M		kg/kmol	Molecular weight
	298	T		K	Temperature
	100	p		atm	Pressure
		R	286.987	m^2/(s^2*K)	Gas constant
		ρ	118.478	kg/m^3	Density
	1	UT		K	Temperature uncertainty
		Up	0.370665	atm	Pressure uncertainty
	.5	U		%	Density uncertainty percentage

- Connect the TK model with Excel, using the Wizard.

Example 2: Shaft Torsion Revisited

- Enter the following rule to solve for the stress on a hollow shaft under torsional load:

Rule
Txy = torque*OD/(2*pi()*(OD^4-ID^4)/32)

- Switch to the Variable Sheet and enter the inputs shown below. Time permitting, rearrange the rows, and add the comments:

St	Input	Name	Output	Unit	Comment
	60000	torque		lbf*in	torsional moment
	2	OD		in	outside diameter
	1	ID		in	inside diameter
		Txy	40743.7	lbf/in^2	torsional shear stress

- Now estimate the uncertainty in the resulting Txy value by adding the following rules. You will find it easiest to enter the first rule, then copy and paste the next two and edit them accordingly.

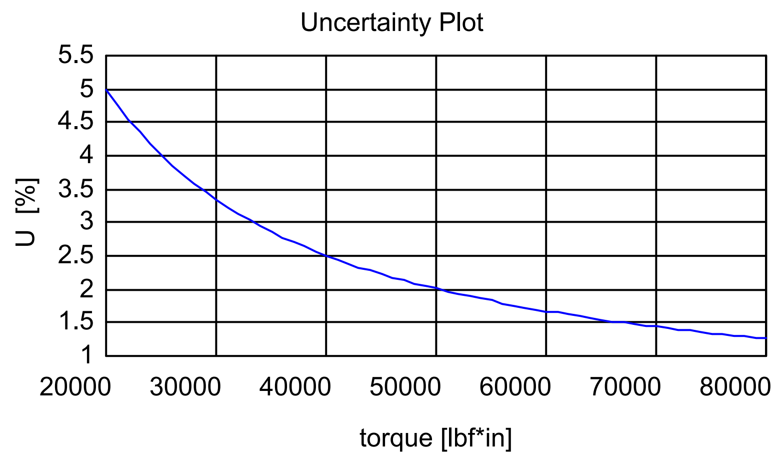
Rule
u1 = UOD*deriv("torque*OD/(2*pi()*(OD^4-ID^4)/32),OD",OD)/Txy
u2 = UID*deriv("torque*OD/(2*pi()*(OD^4-ID^4)/32),ID",ID)/Txy
u3 = Utorque*deriv("torque*OD/(2*pi()*(OD^4-ID^4)/32),torque",torque)/Txy
U = sqrt(u1^2 + u2^2 + u3^2)*100

- Enter the inputs shown below and solve.

St	Input	Name	Output	Unit	Comment
	60000	torque		lbf*in	torsional moment
	2	OD		in	outside diameter
	1	ID		in	inside diameter
		Txy	40743.7	lbf/in^2	torsional shear stress
	1000	Utorque		lbf*in	Uncertainty in torque
	.001	UOD		in	Uncertainty in OD
	.001	UID		in	Uncertainty in ID
		U	1.675	%	Percent uncertainty in computing Txy
		u1	-1.633E-3		OD uncertainty component
		u2	2.667E-4		ID uncertainty component
		u3	1.667E-2		torque uncertainty component

- Use the TK Solver List Solve Wizard to generate a plot of U vs. torque as torque varies from 20000 to 80000 in steps of 2000.

Here is the plot.



Here is a similar plot showing the relationship between the uncertainty in the stress and the OD as the OD varies from 0.001 to 0.01 in and a constant torque of 60000 lbf*in.