Time-Step Solutions: Tank Draining

This example is taken from an excellent book by Johannes Gessler of Colorado State University, entitled TK Solver -- A Tutorial, published by McGraw-Hill. The book features many enlightening exercises and we encourage anyone wishing to hone their TK skills to purchase a copy.

Water flows from a tank through a pipe and discharges into air at its end. How long does it take for the water level (H) to go from 50 ft to 20 ft?

In effect, we are solving the differential equation q*dt = a*dH , where **a** is the horizontal area of the tank, **dH** is the change in the water level, **q** is the flow rate, and **dt** is the increment of time. We also know:

1. The friction factor equation is f = (-2 * log(e/(3.7*d) + 2.51*nu/(v*d*sqrt(f))))^-2
2. The head loss equation is H = f*L/d*v^2/(2*g)
3. The flow rate equation is q = .25*v*pi()*d^2
4. The drain pipe has length (L) 2000 feet and diameter (d) of 8 inches (2/3 ft)
5. The acceleration due to gravity (g) is 32.2 ft/s^2
6. The absolute roughness of the pipe (e) is .00085 ft
7. The kinematic viscosity (nu) is .00001 ft^2/s
8. The horizontal area (a) is 500 ft^2.

Solution:

- Enter the three equations above for f, H, and q on the TK Rule Sheet.

- Rearrange the Variable Sheet as shown below and enter inputs for the known values as described above. (An equation involving the variable **a** will be entered later. Ignore it for now.)

Since **f** is an unknown appearing twice in the first equation, a guess is necessary to solve for it.

- Open the subsheet for **f** and enter a default first guess of .01 (f is typically between .01 and .03). Then, close the subsheet and solve.

Sta	Input	Name	Output	Unit	Comment
	50	H		ft	water level in the tank
		f	.021384747		friction factor (default guess set to .01)
	2000	L		ft	length of pipe
	.666666667	d		ft	diameter of pipe
		v	7.08	ft/s	average velocity in pipe
	32.2	g		ft/s^2	gravity
	.00085	e		ft	absolute roughness
	.00001	nu		ft^2/s	kinematic viscocity
		q	2.47	ft^3/s	flow rate

At a given water level, we are able to compute the corresponding flow information.

- Add the following equations to the Rule Sheet. These represent the differential equation to be solved.

```
i = elt()
if i <= 1 then dt = 0  else  (q+'q[i-1])/2*dt = a*('H[i-1]-H)   ;  change in volume
```

The first rule uses the built-in ELT function to return the current time step number during solving or list solving. ELT = 0 during direct solving and returns the current element number during list solving.

The second rule solves for the time change, initially setting it to 0, with subsequent computations done by dividing the changes in volume by the *average* flow rates during each of the time steps.

Note that without the if condition, TK would fail during the first element of list solving due to a reference to a 0th element of a list.

- Add an equation to sum the individual time changes for the total draining time.

```
if i <= 1 then place(‘total,1) = 0 else place(‘total,i) = 'total[i-1] + dt
```

Again, the conditional expression is used to avoid an error during the initial time step. The built-in place function is used to send the total time value at each step to a list.

- Set up the Variable Sheet for list solving. **H**, **v**, **q**, and **dt** are associated with lists and **H** is the input. The variable **a** now appears at the bottom of the Variable Sheet. Input the value 500 for **a**.

Sta	Input	Name	Output	Unit	Comment
L	50	H		ft	water level in the tank
		f	.021384747		friction factor (default guess set to .01)
	2000	L		ft	length of pipe
	.666666667	d		ft	diameter of pipe
L		v	7.08	ft/s	average velocity in pipe
	32.2	g		ft/s^2	gravity
	.00085	e		ft	absolute roughness
	.00001	nu		ft^2/s	kinematic viscocity
L		q	2.47	ft^3/s	flow rate
L		dt	0	s	time step size
	500	a		ft^2	horizontal area of tank

- Using the List Fill command, fill the H list with values from 50 to 20 in steps of -1.

- List Solve and create a formatted summary table of the results.

Element	H	v	q	dt	total
1	50	7.08	2.47	0	0
2	49	7.01	2.45	203.22	203.22
3	48	6.94	2.42	205.33	408.55
4	47	6.87	2.4	207.5	616.05
5	46	6.79	2.37	209.75	825.8
6	45	6.72	2.34	212.07	1037.88
7	44	6.64	2.32	214.47	1252.35
8	43	6.56	2.29	216.96	1469.31
9	42	6.49	2.26	219.53	1688.83
10	41	6.41	2.24	222.19	1911.03
11	40	6.33	2.21	224.95	2135.98
12	39	6.25	2.18	227.82	2363.8
13	38	6.17	2.15	230.8	2594.61
14	37	6.08	2.12	233.9	2828.51
15	36	6	2.09	237.13	3065.64
16	35	5.91	2.06	240.49	3306.13
17	34	5.83	2.03	244	3550.14
18	33	5.74	2	247.67	3797.81
19	32	5.65	1.97	251.51	4049.32
20	31	5.56	1.94	255.53	4304.85
21	30	5.47	1.91	259.75	4564.6
22	29	5.38	1.88	264.18	4828.79
23	28	5.28	1.84	268.85	5097.64
24	27	5.18	1.81	273.77	5371.41
25	26	5.09	1.78	278.97	5650.39
26	25	4.98	1.74	284.48	5934.87
27	24	4.88	1.7	290.33	6225.2
28	23	4.78	1.67	296.55	6521.74
29	22	4.67	1.63	303.18	6824.92
30	21	4.56	1.59	310.28	7135.2
31	20	4.45	1.55	317.89	7453.09

Using 1 inch step sizes for H, the resulting drain time total is computed as 7453.09. How accurate is this and what are the effects of different step sizes? The worst estimate we could get would be to have a single step of 30 inches, in which case dt is computed as 500*30/2.0131645 = 7450.96. 1000 steps results in 7453.09. We should feel confident.