Simplifying can make things complex

Consider the equation

$$x = r^{\left[\frac{k-1}{k}\right]} - 1$$

If we know values of r and k, we can easily compute x. What if we know x and r and need to compute k? No problem... we can use a symbolic algebra software package to derive the equivalent equation in terms of k.

$$k = \frac{\ln(r)}{\ln(r) - \ln(x+1)}$$

What if we need to solve for r? Again, we can simplify using algebra software.

$$r = (x+1)^{\left[\frac{k}{k-1}\right]}$$

We now have equations that solve for each of the three variables in the original equation. If we enter all three on the TK Solver rule sheet, TK can directly solve the equation with the isolated unknown. Let's try them out with the following inputs… r = 2 and k = 1 .

The first equation solves directly for x = 0.

The second equation is found consistent, saying that 1 = ln(2)/ln(2).

The third equation fails, with TK reporting a division by zero error message when TK divides 1 by (1-1).

One way to avoid this error is to use a conditional rule

$$\text{if } k <> 1 \text{ then } r = (x+1)^{\left[\frac{k}{k-1}\right]}$$

The simplest approach of all is to use a single equation where each of the variables appears just once.

$$x = r^{\left[1 - \frac{1}{k}\right]} - 1$$

TK's Direct Solver can solve this equation given any two of the three variables, even for the troublesome input of k = 1. The only condition that will cause an error is when k = 0, but that is the case for all the other equations as well.