

Example

A company produces rubber used for tires by combining three ingredients: rubber, oil, and carbon black. The cost of rubber is 0.04 per pound. The cost of oil is 0.01 per pound. The cost of carbon black is 0.07 per pound. Here are the equations relating the composition with its characteristics.

$$TS = 12.5 - 0.1 \cdot \text{Oil} - 0.001 \cdot \text{Oil}^2$$

$$E = 17 + 0.35 \cdot \text{Rubber} - 0.04 \cdot \text{Oil} - 0.002 \cdot \text{Rubber}^2$$

$$H = 34 + 0.1 \cdot \text{Rubber} + 0.06 \cdot \text{Oil} - 0.3 \cdot \text{Carbon} + 0.001 \cdot \text{Rubber} \cdot \text{Oil} + 0.005 \cdot \text{Oil}^2 + 0.001 \cdot \text{Carbon}^2$$

The hardness must be between 25 and 35, the elasticity must be at least 16. The tensile strength must be at least 12.

Here is the equation for the cost of the mixture.

$$\text{\$Oil} \cdot \text{Oil} + \text{\$Rubber} \cdot \text{Rubber} + \text{\$Carbon} \cdot \text{Carbon} = \text{Cost}$$

Another equation is added to compute the total pounds in the mixture.

$$\text{Weight} = \text{Oil} + \text{Rubber} + \text{Carbon}$$

Here is the Variable Sheet with some sample inputs and outputs.

St	Input	Name	Output	Unit	Comment
	20	Oil		lb	Oil in the mix
	40	Rubber		lb	Rubber in the mix
	40	Carbon		lb	Carbon in the mix
		Weight	100	lb	Total mix
	.01	\\$Oil		\$	Oil cost per pound
	.04	\\$Rubber		\$	Rubber cost per pound
	.07	\\$Carbon		\$	Carbon black cost per pound
		Cost	4.6	\$	Cost/pound of mixture
		TS	10.1		Tensile strength
		E	27		Elasticity
		H	31.6		Hardness

Using the equations and variables above, the model will now solve for the cost of the material for a given mixture. The Optimizer can be set up to solve for the mixture producing the required qualities at the lowest cost.

The target variable is Cost, and it should be minimized. The change variables are Oil, Rubber, and Carbon, with lower bounds set as 0, 25, and 50. Upper bounds are set as

100, 60, and 100. These bounds are based on the assumption that Weight is 100. Here are the constraints.

Weight = 100
 TS => 12
 E => 16
 H => 25
 H <= 35

The Optimizer (F11) returns the following solution.

St	Input	Name	Output	Unit	Comment
	4.77275583	Oil		lb	Oil in the mix
	45.2272442	Rubber		lb	Rubber in the mix
	50	Carbon		lb	Carbon in the mix
		Weight	100	lb	Total mix
	.01	\$Oil		\$	Oil cost per pound
	.04	\$Rubber		\$	Rubber cost per pound
	.07	\$Carbon		\$	Carbon black cost per pound
		Cost	5.35681733	\$	Cost/pound of mixture
		TS	11.9999452		Tensile strength
		E	28.547618		Elasticity
		H	26.6388444		Hardness

This example model is located in the Optimization section of the TK Library, in the Optimizer Examples group.