Optimization Case Study – Welded Beam Design

This example is taken from Engineering Optimization, Methods and Applications by G.V. Reklaitis, A Ravindran, and K. M. Ragsdell, published in 1983 by John Wiley & Sons.

A beam is to be welded to a rigid support member. The beam must support a force of 6000 lbf. The beam dimensions are to be selected so that the system cost is minimized.

Here is a summary of the variables included in the math model, with the solution values.

St	Input	Name	Output	Unit	Comment
					Welded Beam
	14	L		in	Beam length beyond weld
	0.244369	x1		in	Height of the weld
	6.21861	x2		in	Length of the weld
	8.29147	x3		in	Depth of welded beam
	0.244369	x4		in	Width of welded beam
	6000	F		lbf	Force on beam
	13600	taud		psi	Design shear stress
	30000	sigd		psi	Design normal stress for beam material
		M	102656	in*lbf	Moment of F about the center of gravity of the weld
		J	46.0646	in^4	Polar moment of inertia of the weld group
		tau1	2791.88	psi	Primary stress on weld throat
		tau2	11767.5	psi	Secondary torsional stress on weld
		taux	13600	psi	Weld stress
		y	1.98702	$	Cost
					Constraints
		c1	0.0000374865		
		c2	0.0000330776		
		c3	0		
		c4	0.0000107034		
		c5	0.119369		
		c6	0.234241		

The equations are displayed on the following page.

$$y = (1 + 0.37 \cdot 0.283) \cdot x1^{3} + 0.17 \cdot 0.283 \cdot x3 \cdot x4 \cdot (L + x2)$$

$$tau1 = \frac{F}{\sqrt{2} \cdot x1 \cdot x2}$$

$$M = F \cdot \left[L + \frac{x2}{2} \right]$$

$$R = \sqrt{\frac{x2^{2}}{4} + \frac{(x3 + x1)^{2}}{4}}$$

$$J = 2 \cdot \left[0.707 \cdot x1 \cdot x2 \cdot \left[\frac{x2^{2}}{12} + \frac{(x3 + x1)^{2}}{4} \right] \right]$$

$$tau2 = \frac{M \cdot R}{J}$$

$$taux = \sqrt{tau1^{2} + \frac{2 \cdot tau1 \cdot tau2 \cdot x2}{2 \cdot R} + tau2^{2}}$$

$$c1 = taud - taux$$

$$c2 = sigd - \frac{6 \cdot F \cdot L}{x4 \cdot x3^{2}}$$

$$c3 = x4 - x1$$

$$E = 3E7$$

$$I = \frac{1}{12} \cdot x3 \cdot x4^{3}$$

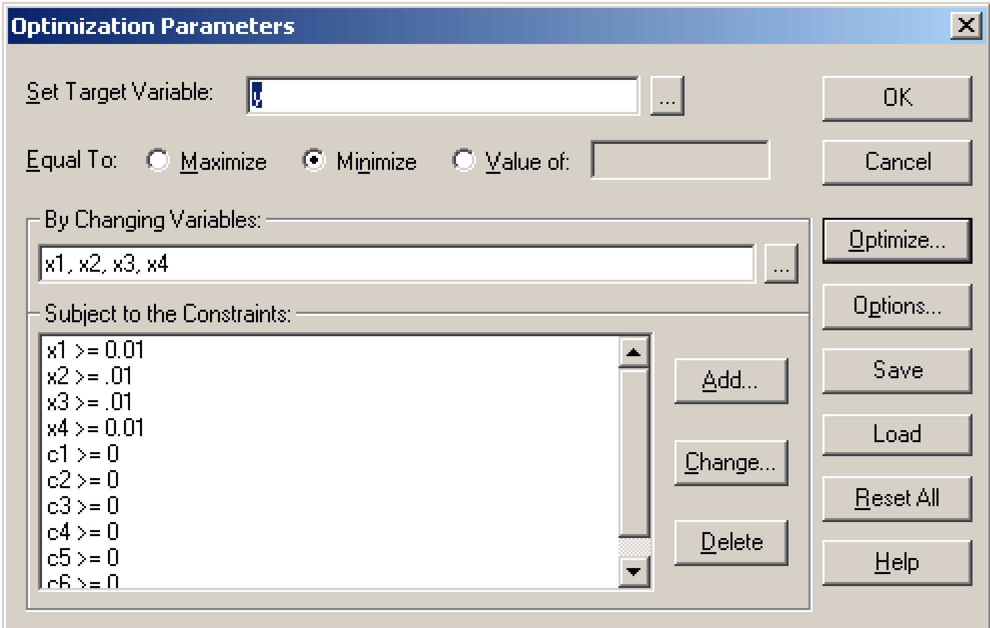
$$a = \frac{1}{3} \cdot 12E6 \cdot x3 \cdot x4^{3}$$

$$c4 = \frac{4.013 \cdot \sqrt{E \cdot I \cdot a}}{L^{2}} \cdot \left[1 - \frac{x3}{2 \cdot L} \cdot \sqrt{\frac{E \cdot I}{a}} \right] - F$$

$$c5 = x1 - 0.125$$

$$c6 = 0.25 - \frac{4 \cdot F \cdot L^{3}}{[illegible]^{3}}$$

The Optimization Parameters are concisely defined as follows.


Optimization Parameters
Set Target Variable:
Equal To:
Maximize
Minimize
Value of:
By Changing Variables:
x1, x2, x3, x4
Subject to the Constraints:
x1 >= 0.01
x2 >= .01
x3 >= .01
x4 >= 0.01
c1 >= 0
c2 >= 0
c3 >= 0
c4 >= 0
c5 >= 0
Add...
Change...
Delete
OK
Cancel
Optimize...
Options...
Save
Load
Reset All
Help