Optimization Case Study – Pipeline Transmission System

This example is taken from Engineering Optimization, Methods and Applications by G.V. Reklaitis, A Ravindran, and K. M. Ragsdell, published in 1983 by John Wiley & Sons.

A natural gas pipeline transmission system is required to pump 100 MMSCF/day of natural gas through a distance of 600 miles. Compressor stations are to be placed at equal distances. The design variables are the pipe diameter D, compressor discharge and inlet pressures P1 and P2, and length between stations L. The optimum design should be such that the total cost of the pipeline facility is minimized.

Here are the equations.

$$R = 1.987$$

$$r = \frac{P1}{P2}$$

$$X = r^{0.219} - 1$$

$$hp = 27168 \cdot X$$

$$Cfixed = \frac{4.5E6}{L} + \frac{7.08E8}{L} \cdot X$$

$$Cfuel = \frac{5.82E8}{L} \cdot X$$

$$Cmain = \frac{0.58E8}{L} \cdot X$$

$$Clabor = \frac{8.64E6}{L} + \frac{1.96E8}{L} \cdot X$$

$$Pinstall = 73800 \cdot D$$

$$t = \frac{D \cdot P1}{50000}$$

$$Pmatl = 15.68 \cdot D^2 \cdot P1$$

$$T = Pmatl + 9E5 + Pinstall + Cfixed + Cfuel + Cmain + Clabor$$

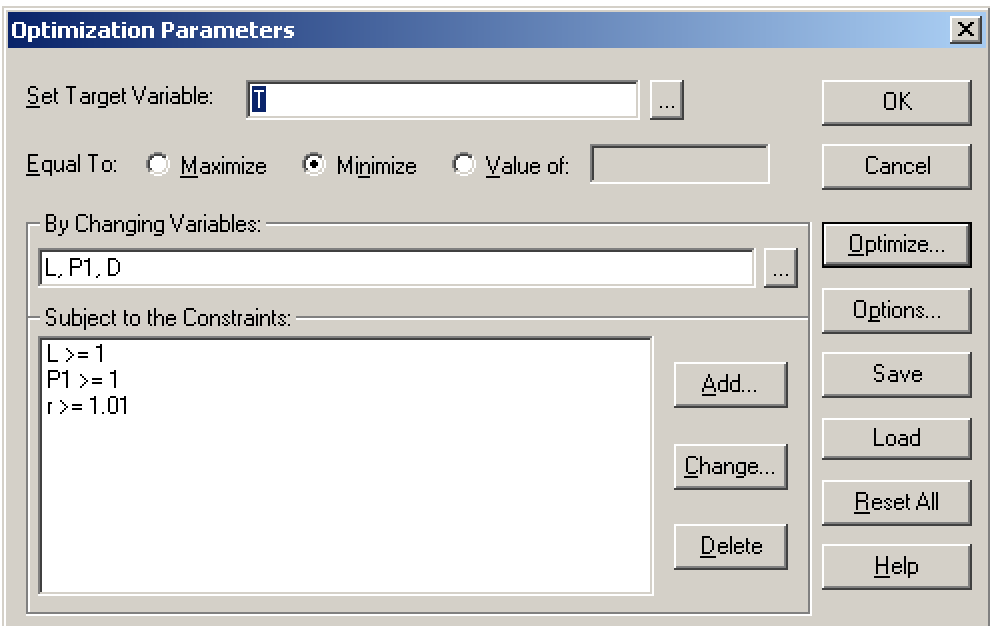
$$f = 0.008 \cdot D^{\left[\frac{-1}{3}\right]}$$

$$Q = 3.39 \cdot \sqrt{\frac{\frac{\left[P1^2 - P2^2\right] \cdot \left[D^5\right]}{f}}{L}}$$

Here is a summary of the variables, with descriptions and solution values.

St	Input	Name	Output	Unit	Comment
	1,405,790	Q		SCF/hr	Volumetric flow
	88.2048	L		mi	Length of pipe between compressors
	14.9774	D		in	Pipe diameter
		t	0.141202	in	Pipe thickness
	471.382	P1		psia	Compressor discharge pressure
		P2	396.087	psia	Compressor inlet pressure
		r	1.1901		P1/P2
		hp	1,055.46	hp	Horsepower of each compressor
		Cfixed	362,853	$	Annual fixed cost of all compressors
		Cfuel	256,339	$	Annual fuel cost
		Cmain	25,545.8	$	Annual maintenance cost
		Clabor	184,281	$	Annual labor cost
		Pinstall	1,105,340	$	Pipe installation cost
		Pmatl	1,658,040	$	Annual cost of pipe material
		T	4,492,390	$	Total annual cost of operation

The TK Solver Optimizer is concisely configured as follows.



Here is a TK table containing the optimal solutions for a range of flow rates.

Element	Q	L	D	P1	T
1	10,000,000	35.6628	37.0435	190.584	9,784,880
2	9,000,000	37.4409	35.2846	200.092	9,363,160
3	8,000,000	39.5325	33.4177	211.269	8,915,380
4	7,000,000	42.0454	31.4203	224.7	8,436,300
5	6,000,000	45.1458	29.2622	241.275	7,918,750
6	5,000,000	49.1369	26.8952	262.609	7,352,300
7	4,000,000	54.4442	24.2651	291.021	6,720,860
8	3,000,000	62.1849	21.2467	332.413	5,997,100
9	2,000,000	74.9624	17.624	400.602	5,127,170
10	1,000,000	103.22	12.799	551.595	3,969,820

The table was created in a few simple steps. First, each of the variables in the table was assigned the List status on the variable sheet. The flow rate was input and the Optimizer run. The results were then placed into lists using TK's Put in Lists Command. These steps were repeated for each of the desired flow rates.

The lists can also be plotted, as shown below.