

TK Solver Optimization Case Study

A certain corporation is planning to produce and market three different products. Let x_1 , x_2 , and x_3 denote the number of units of the three respective products to be produced. The preliminary estimates of their potential profitability are as follows.

For the first 15 units produced of product 1, the unit profit would be approximately \$36. The unit profit would only be \$3 for any additional units of product 1. For the first 20 units produced of product 2, the unit profit is estimated to be \$24. The unit profit would only be \$12 for each of the next 20 units, and \$9 for any additional units. For the first 10 units of product 3, the unit profit would be \$45. The unit profit would be \$30 for each of the next 5 units and \$18 for any additional units.

Certain limitations on the use of the needed resources impose the following constraints on the production of the three products:

$$\begin{aligned} 60 &\Rightarrow x_1 + x_2 + x_3 \\ 200 &\Rightarrow 3x_1 + 2x_2 \\ 70 &\Rightarrow x_1 + 2x_3 \end{aligned}$$

There is another constraint that the profit from products 1 and 2 must total at least \$900. Here are the TK rules for the profits on product 1.

Rule
if $x_1 < 0$ then $p_1 = 0$
if and($x_1 \geq 0, x_1 \leq 15$) then $p_1 = 36x_1$
if $x_1 > 15$ then $p_1 = 540 + 3(x_1 - 15)$

The variable p_1 is the total profit from product 1. The same types of rules can be used for products 2 and 3.

Rule
if $x_2 < 0$ then $p_2 = 0$
if and($x_2 \geq 0, x_2 \leq 20$) then $p_2 = 24x_2$
if and($x_2 > 20, x_2 \leq 40$) then $p_2 = 480 + 12(x_2 - 20)$
if $x_2 > 40$ then $p_2 = 720 + 9(x_2 - 40)$
if $x_3 < 0$ then $p_3 = 0$
if and($x_3 \geq 0, x_3 \leq 10$) then $p_3 = 45x_3$
if and($x_3 > 10, x_3 \leq 15$) then $p_3 = 450 + 30(x_3 - 10)$
if $x_3 > 15$ then $p_3 = 600 + 18(x_3 - 15)$

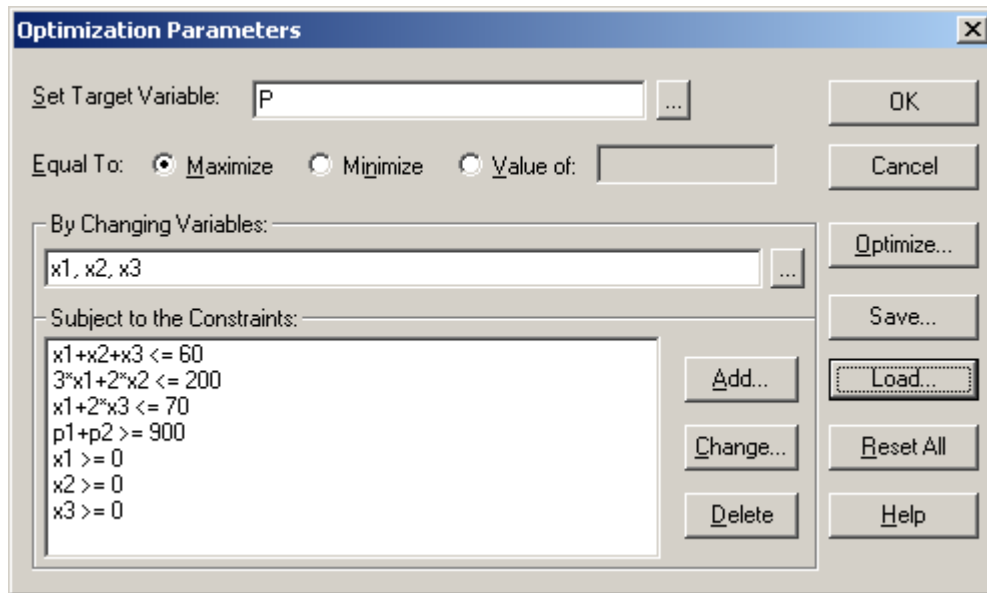
Rules are added for the total profit and the constraints.

Rule
$P = p_1 + p_2 + p_3$
$c_1 = x_1 + x_2 + x_3$
$c_2 = 3x_1 + 2x_2$
$c_3 = x_1 + 2x_3$
$c_4 = p_1 + p_2$

Here is the TK Variable Sheet summarizing the variables.

St	Input	Name	Output	Unit	Comment
	22	x1			Units of product 1
	18	x2			Units of product 2
	20	x3			Units of product 3
		p1	561		Profit from product 1
		p2	432		Profit from product 2
		p3	690		Profit from product 3
		P	1683		Total profit
					Constraint variables:
		c1	60		≤ 60
		c2	102		≤ 200
		c3	62		≤ 70
		c4	993		≥ 900

The TK Optimizer is then invoked and the parameters are set.



When the Optimize button is clicked, TK returns the solution on the variable sheet.

St	Input	Name	Output	Unit	Comment
	15	x1			Units of product 1
	20	x2			Units of product 2
	25	x3			Units of product 3
		p1	540		Profit from product 1
		p2	480		Profit from product 2
		p3	780		Profit from product 3
		P	1800		Total profit