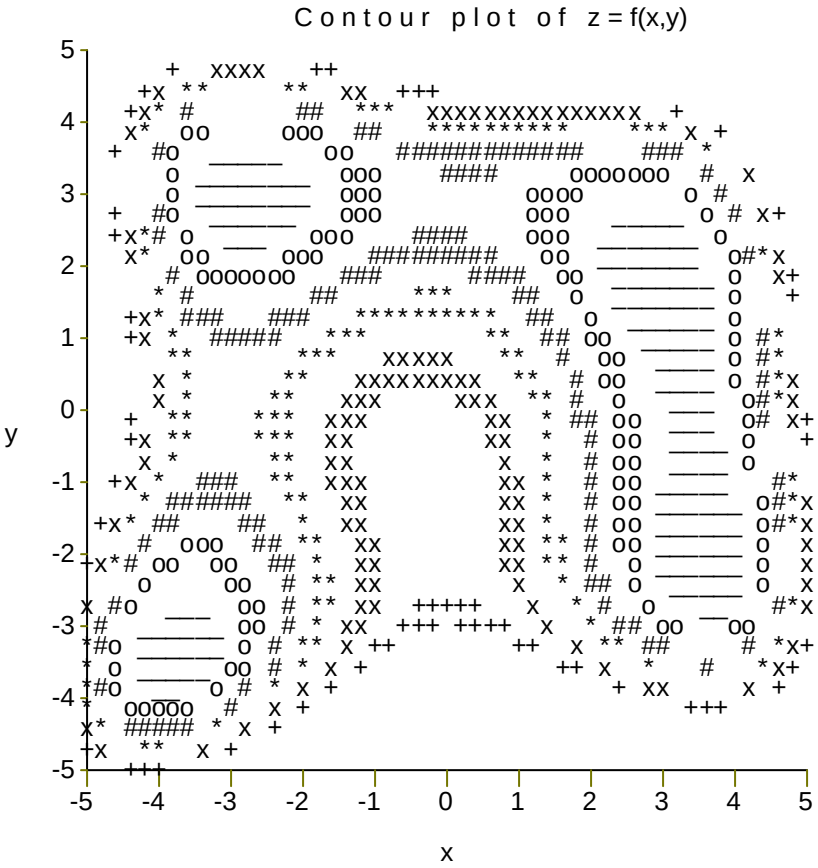
Optimization Case Study

We will investigate an equation known as Himmelblau's function.

$$z = \left[x^2 + y - 11\right]^2 + \left[x + y^2 - 7\right]^2$$

This equation is used in the Graphics Utilities section TK Library as an example for creating contour plots. Here is the plot of the function.



The low areas are indicated with "_" symbols and the high areas are shown with "+" symbols. The complete symbol key is shown in the CONTOUR_doc comment.

```
   0 -  9.1%   _          54.5 - 63.6%   *
 9.1 - 18.2%   blank      63.6 - 72.2%   blank
18.2 - 27.3%   o          72.2 - 81.8%   x
27.3 - 36.4%   blank      81.1 - 90.9%   blank
36.4 - 45.5%   #          90.9 -  100%   +
45.5 - 54.5%   blank
```

It's easy to see by the plot that there are four main areas where the function hits local minimums, moving clockwise -- (3,2), (3.5,-2), (-4,-3.5), (-2.5,3).

We can use the TK Optimizer to find these points.

1. Enter a rule on the rule sheet to reference the Himmel function.

   z = Himmel(x,y)

2. Enter trial values for x and y on the variable sheet and solve.

St	Input	Name	Output	Unit	Comment
		z	8.125		
	-3.5	x			
	-3.5	y			

3. Set up the Optimizer, with z as the target to minimize and x and y as the change variables. Here is the solution.

St	Input	Name	Output	Unit	Comment
		z	1.3737E-14		
	-3.7793103	x			
	-3.283186	y			

This process can be repeated in all four regions indicated by the contour plot.

St	Input	Name	Output	Unit	Comment
		z	8.0319E-14		
	3.00000005	x			
	1.99999998	y			

St	Input	Name	Output	Unit	Comment
		z	1.734E-12		
	-2.8051182	x			
	3.1313127	y			

St	Input	Name	Output	Unit	Comment
		z	5.4166E-12		
	3.58442855	x			
	-1.848127	y			