Optimization Case Study

The following problem is taken from "Basic Programs for Production and Operation Management" by Pantumsinchai, Hassan, and Gupta (Prentice-Hall, 1983).

Butane, Virgin Naphta and Catalytic Cracked Gasoline are to be blended into Super and Regular unleaded gasoline. The goal is to blend the components in such a way as to produce the two types of gasoline with a maximum profit. There are six unknowns corresponding with the daily consumption of components used for each kind of gasoline. This can be represented by two equations, with TS and TR representing the total units of Super and Regular produced. For example, the variable x11 represents the units of Butane in Super.

$$TS = x11 + x21 + x31$$
$$TR = x12 + x22 + x32$$

For any blend, the octane can be computed for Super and Regular using the following equations, with OCT1, OCT2, and OCT3, the octane values of the three components.

$$OCTS = \frac{OCT1 \bullet x11 + OCT2 \bullet x21 + OCT3 \bullet x31}{TS}$$
$$OCTR = \frac{OCT1 \bullet x12 + OCT2 \bullet x22 + OCT3 \bullet x32}{TR}$$

The minimum octane numbers for Super and Regular are 92 and 87 respectively.

Another important criterion of the blends is the Reid Vapor Pressure (RVP). The following equations are used to compute the RVP for Super and Regular,with RVP1, RVP2, and RVP3 the values from each of the components.

$$RVPS = \frac{RVP1 \bullet x11 + RVP2 \bullet x21 + RVP3 \bullet x31}{TS}$$
$$RVPR = \frac{RVP1 \bullet x12 + RVP2 \bullet x22 + RVP3 \bullet x32}{TR}$$

The maximum for both RVPS and RVPR is 8.

There are limitations on the availability of each of the components. The equations for the total of each component used are as follows.

$$A1 = x11 + x12$$
$$A2 = x21 + x22$$
$$A3 = x31 + x32$$

This will allow us to compare A1, A2, and A3 with the available amounts.

The last equation defines the total profit from both blends, where p1 and p2 are the profits per unit of production of Super and Regular, respectively.

$$F = p1 \bullet TS + p2 \bullet TR$$

These equations can be entered into TK Solver to establish the objective function for the optimization. The TK Solver Variable Sheet is shown next.

St	Input	Name	Output	Unit	Comment
					Blending Gasoline
	12	RVP1			Butane Vapor Pressure
	9	RVP2			Virgin Naphta Vapor Pressure
	6	RVP3			C/C Gasoline Vapor Pressure
	120	OCT1			Butane Octane
	100	OCT2			Virgin Naphta Octane
	80	OCT3			C/C Gasoline Octane
	3	p1			Profit per unit of Super
	1	x11			Butane in Super
	1	x21			Virgin Naphta in Super
	1	x31			C/C Gasoline in Super
	2.4	p2			Profit per unit of Regular
	1	x12			Butane in Regular
	1	x22			Virgin Naphta in Regular
	1	x32			C/C Gasoline in Regular
		A1	2		Units of Butane
		A2	2		Units of Virgin Naphta
		A3	2		Units of C/C Gasoline
		RVPS	9		Reid Vapor Pressure of Super
		RVPR	9		Reid Vapor Pressure of Regular
		OCTS	100		Octane of Super
		OCTR	100		Octane of Regular
		TS	3		Total Units of Super
		TR	3		Total Units of Regular
		F	16.2	$	Total Profit

Inputs are provided for each of the components and with values of 1 input for each of the six sample quantities, we see the results at the bottom of the sheet. Our goal is to determine the values of these six quantities which result in a maximum value of F and which satisfy the vapor pressure and octane requirements for the blends. To accomplish this, we can use the TK Optimizer.

We launch the Optimizer and define the objective function and the constraints.

- Total profit, variable F, is selected as the objective variable.

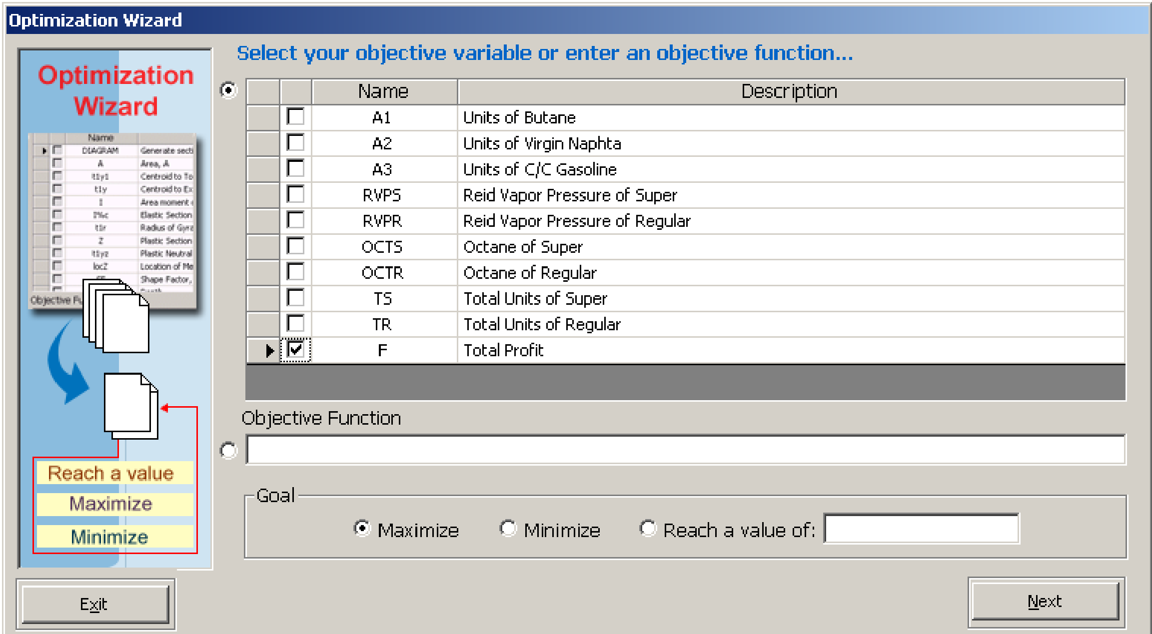
- Quantities x11, x21, x31, x12, x22, x32 are identified as decision variables with minimal values set to 0.

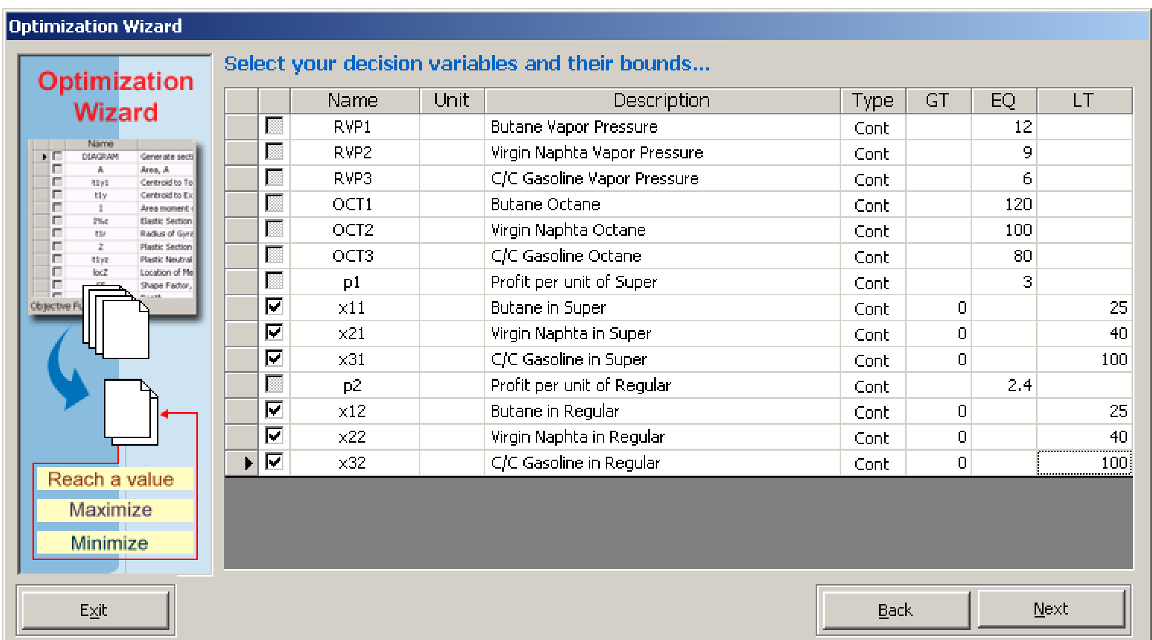
- Constraints are made on the octane of super and regular, variables OCTS and OCTR, to be a minimum of 92 and 87, respectively.

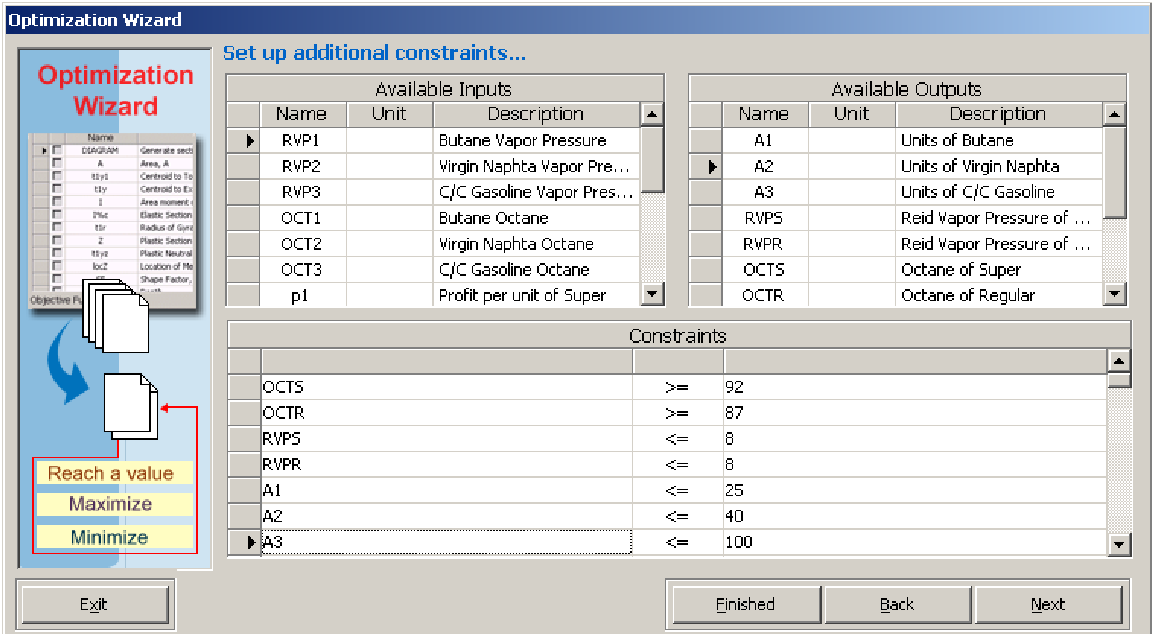
- Constraints are made on the resulting vapor pressures of both the Super and Octane blends. In this case, the maximums are 8 for both.

- Constraints are made on the available units of each of the three components. In this case, there are 25 units of Butane, 40 units of Virgin Naptha, and 100 units of C/C Gasoline.

The Optimization Wizard screens are shown on the following pages, along with the optimal solution.


Optimization Wizard
Select your objective variable or enter an objective function...
Optimization Wizard
Name | Description
A1 | Units of Butane
A2 | Units of Virgin Naphta
A3 | Units of C/C Gasoline
RVPS | Reid Vapor Pressure of Super
RVPR | Reid Vapor Pressure of Regular
OCTS | Octane of Super
OCTR | Octane of Regular
TS | Total Units of Super
TR | Total Units of Regular
F | Total Profit
Objective Function
Goal
Maximize
Minimize
Reach a value of:
Reach a value
Maximize
Minimize
Exit
Next


Optimization Wizard
Select your decision variables and their bounds...
Optimization Wizard
Name | Unit | Description | Type | GT | EQ | LT
RVP1 | Butane Vapor Pressure | Cont | 12
RVP2 | Virgin Naphta Vapor Pressure | Cont | 9
RVP3 | C/C Gasoline Vapor Pressure | Cont | 6
OCT1 | Butane Octane | Cont | 120
OCT2 | Virgin Naphta Octane | Cont | 100
OCT3 | C/C Gasoline Octane | Cont | 80
p1 | Profit per unit of Super | Cont | 3
x11 | Butane in Super | Cont | 0 | 25
x21 | Virgin Naphta in Super | Cont | 0 | 40
x31 | C/C Gasoline in Super | Cont | 0 | 100
p2 | Profit per unit of Regular | Cont | 2.4
x12 | Butane in Regular | Cont | 0 | 25
x22 | Virgin Naphta in Regular | Cont | 0 | 40
x32 | C/C Gasoline in Regular | Cont | 0 | 100
Reach a value
Maximize
Minimize
Exit
Back
Next



When the "Finished" button is clicked, the solution is quickly displayed on the TK Variable Sheet.

St	Input	Name	Output	Unit	Comment
	12	RVP1			Butane Vapor Pressure
	9	RVP2			Virgin Naphtha Vapor Pressure
	6	RVP3			C/C Gasoline Vapor Pressure
	120	OCT1			Butane Octane
	100	OCT2			Virgin Naphtha Octane
	80	OCT3			C/C Gasoline Octane
	3	p1			Profit per unit of Super
	18.7	x11			Butane in Super
	40	x21			Virgin Naphtha in Super
	70.3	x31			C/C Gasoline in Super
	2.4	p2			Profit per unit of Regular
	6.3	x12			Butane in Regular
	0	x22			Virgin Naphtha in Regular
	29.7	x32			C/C Gasoline in Regular
		A1	25		Units of Butane
		A2	40		Units of Virgin Naphtha
		A3	100		Units of C/C Gasoline
		RVPS	7.8		Reid Vapor Pressure of Super
		RVPR	7.05		Reid Vapor Pressure of Regular
		OCTS	92		Octane of Super
		OCTR	87		Octane of Regular
		TS	129		Total Units of Super
		TR	36		Total Units of Regular
		F	473.4		Total Profit