Mathematical Modeling of Families of Charts

12/18/02

TK Solver is used to computerize stress calculations involving spherical shells with radial loads on nozzle connections. Bulletin 107 from the Welding Research Council is the source for the calculations. That document, titled "Local Stresses in Spherical and Cylindrical Shells Due to External Loading", is authored by K.R. Wichman, A.G. Hopper, and J.L. Mershon, with the most recent update in October 2002. The ISSN number is 0043-2326.

This paper summarizes the required calculations and how they are incorporated into a dynamic mathematical model. Issues such as interpolation within and between charts, solution of simultaneous equations, and unit conversions are stressed.

The combined stress formulas used are as follows.

$$sigxCr = Km \cdot sigxM + Kb \cdot sigxB$$
$$sigyCc = Km \cdot sigyM + Kb \cdot sigyB$$

Variable sigxCr represents the normal compressive stress in the radial direction and sigyCc represents the normal compressive stress in the circumferential direction.

In variables sigxM, sigxB, sigyM, and sigyB, the x component represents radial stress while the y component represents tangential stress. The M component represents membrane stress and the B component represents bending stress. So, sigxM can be interpreted as radial membrane stress.

$$sigxM = \frac{Nx}{T}$$
$$sigyM = \frac{Ny}{T}$$

$$sigxB = \frac{6 \cdot Mx}{T^2}$$
$$sigyB = \frac{6 \cdot My}{T^2}$$

Variable T is the shell thickness. N and M represent membrane force and bending moment. These factors must be retrieved from a family of charts discussed below.

In the combined stress formulas, Km and Kb are stress concentration factors defined by the following formulas.

$$Kb = 1 + \left[\frac{1}{\frac{9.4 \bullet rf}{T}} \right]^{0.80}$$

$$Km = 1 + \left[\frac{1}{\frac{5.6 \bullet rf}{T}} \right]^{0.65}$$

The variable rf is the fillet radius between the shell and the attachment.

The variables are summarized on the TK Solver variable sheet, with a sample solution shown in the sheet below.

St	Input	Name	Output	Unit	Comment
					Spherical Shells:
					Cylindrical Attachment Subject to a Radial Load
	30	Rm		in	Mean radius of spherical shell
	0.5	T		in	Thickness of spherical shell
	.125	rf		in	Fillet radius between shell and attachment
	1	r0		in	Outside radius of attachment
	0.875	ri		in	Inner radius of attachment (0 if solid)
F		t	0.125	in	Thickness of attachment
		rm	0.938	in	Mean radius of attachment
		U	0.258199		Shell parameter
	2500	P		lbf	Radial load
		Mx	70.13	in*lbf/in	Bending moment in shell wall in radial direction
		My	397.9	in*lbf/in	Bending moment in shell wall in circumferential direction
		Nx	-362.63	lbf/in	Membrane force in shell wall in radial direction
		Ny	-1741.66	lbf/in	Membrane force in shell wall in circumferential direction
		sigxB	1683	psi	Radial bending stress
		sigyB	9550	psi	Tangential bending stress
		sigxM	-725	psi	Radial membrane stress
		sigyM	-3483	psi	Tangential membrane stress
		Km	1.803557		Membrane stress concentration factor
		Kb	1.504831		Bending stress concentration factor
		sigxCr	1225	psi	Normal compressive stress in the radial direction
		sigyCc	8088	psi	Normal compressive stress in the circumferential direction

As mentioned earlier, the solution requires retrieval of values from families of charts. The source document includes one chart for solid plugs. There are ten charts for hollow nozzles, covering ten combinations of ratios of rm/t and T/t. Here is a table summarizing the ten charts.

Element	rm/t	T/t = 0.25	T/t = 1.0	T/t = 2.0	T/t = 4.0	T/t = 10.0
1	5	SP-1	SP-2	SP-3	SP-4*	
2	15		SP-5	SP-6	SP-7	SP-8*
3	50				SP-9	SP-10

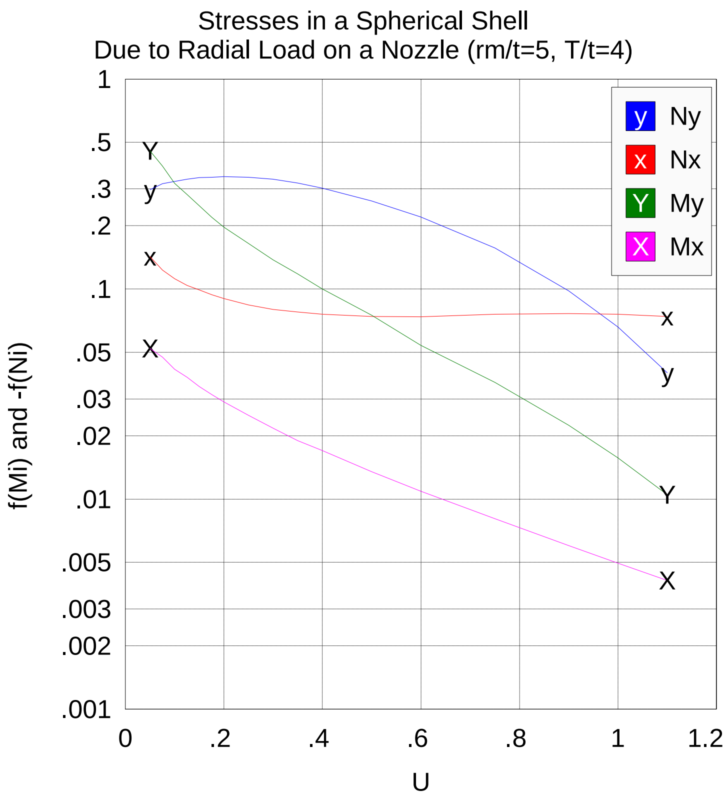
So, for rm/t = 15 and T/t = 2, we get the values of N and M from chart SP-6.

The charts all use a shell parameter, U, on the x-axis. U is defined by the formula

$$U = \frac{r0}{\sqrt{Rm \cdot T}}$$

where r0 is the outer radius of the nozzle and Rm is the outer radius of the shell. Charts SP-4 and SP-8 are asterisked in the table above because they cover a shorter range of U values than the rest of the charts.

Here is chart SP-4, as displayed in TK Solver.



Note that the charts are all semi-log scaled. That is, the y-axis is in log scale and the x-axis is linear. This is important when doing any interpolations between values. For example, assume you know two points (U,X) from one of the curves – (0.4,0.0180) and (1.0,0.00498) -- and you intend to interpolate between them. Given a U value 0.7, linear interpolation would result in a value of X of 0.01149, the point halfway between. A more accurate result is obtained if the interpolation is done using natural logs of the y-axis values and the result is used as the exponent of e. This result is 0.00947. This solution is about 20% different, which is significant when converted to stress.

The TK Solver model includes about 20 data points for each curve on each chart and the interpolations result in very smooth functions. Each of the y-axis values are converted to natural logs before doing any interpolation within TK List Functions. Here is a portion of the List Function for the Ny curve from SP-4. The Domain list contains the U values. The range list contains logs of the y values.

Element	Domain	Range
1	.05	-1.21234105399645
2	.075	-1.14727746064939
3	.1	-1.1239300966524
4	.125	-1.09961278900169
5	.15	-1.07880966137193
6	.175	-1.07734015339257
7	.2	-1.07148362127986
8	.25	-1.07734015339257
9	.3	-1.09961278900169
10	.35	-1.13943428318837
11	.4	-1.19732826160727
12	.5	-1.33750419695046
13	.6	-1.51185758409524
14	.75	-1.85150947363383
15	.9	-2.32278780031157
16	1	-2.71810053695571
17	1.1	-3.2188758248682

The charts and list functions cover 10 discrete combinations. There are actually an infinite number of combinations of rm/t and T/t. Looking at the summary table of SP charts, it is clear that interpolation can be done for values of T/t from 1 to 4 and for values of rm/t from 5 to 15. That is, charts SP-2 to SP-7 form an inclusive block of solutions. In this region, bilinear interpolation can be used to find the results.

Element	rm/t	T/t = 0.25	T/t = 1.0	T/t = 2.0	T/t = 4.0	T/t = 10.0
1	5	SP-1	SP-2	SP-3	SP-4*	
2	15		SP-5	SP-6	SP-7	SP-8*
3	50				SP-9	SP-10

Consider the case where T/t = 1.5 and rm/t = 9. This solution will be bounded by the results from SP-2, SP-3, SP-5, and SP-6. TK Solver includes a bilinear interpolation function which processes the results from each of the charts and combines them as required. In this case, the bilinear interpolation is done assuming the results are in log scale in both the T/t and rm/t directions. That is, it does not assume that the solution for T/t = 1.5 is halfway between the solutions for T/t = 1 and T/t = 2. Rather, the model again converts the ratios to logs before interpolating. As shown earlier, the result is significantly more accurate.

The fact that two of the charts cover an incomplete range of U values presents a problem because you can only interpolate if you have values that bound the point in question. For example, assume that T/t = 3 and rm/t = 10. Looking at the summary table again, this problem is bounded by SP-3, SP-4, SP-6, and SP-7 and bilinear interpolation is possible. However, if U = 1.5, SP-4 is not defined.

Extrapolation can be used to provide solutions where the charts are incomplete. As with interpolation, the extrapolation should be done in log scale for greater accuracy. So, for U = 1.5, an extrapolated value for the SP-4 cell can be obtained by using the results from charts SP-3, SP-6, and SP-7 in log scale to do a linear extrapolation. The formula is

$$\text{if } U > 1.1 \text{ then } {}'spny4_1 = \frac{-e^{\left[\frac{sp3ny(U) \cdot sp7ny(U)}{sp6ny(U)}\right]} \cdot P}{T}$$

This formula places the solution into the 1st row of the 4th column of the bilinear interpolation table in TK. There are similar formulas for all 15 cells of the summary table. There are summary tables for each of the four stress components, Nx, Ny, Mx, and My.

Here is one table.

Element	spny1	spny2	spny3	spny4	spny5
1	-795.712	-893.316	-1171.7	-1696.29	-1935.78
2	-1000.36	-1107	-1292.74	-1819.21	-2058.39
3	-1015.52	-1122.72	-1309.19	-1836.49	-2161.72

The model can now solve over the complete domain of rm/t and T/t ratios, with extrapolation and interpolation where necessary.

Here is the revised summary table showing how each of the cells gets a value.

Element	rm/t	T/t = 0.25	T/t = 1.0	T/t = 2.0	T/t = 4.0	T/t = 10.0
1	5	SP-1	SP-2	SP-3	SP-4*	Extrap.
2	15	Extrap.	SP-5	SP-6	SP-7	SP-8*
3	50	Extrap.	Extrap.	Extrap.	SP-9	SP-10

Thanks to TK Solver, the model can also solve the problem backwards where necessary. For example, a user might want to input an allowable stress and backsolve for the fillet radius on the weld between the shell and the nozzle. Actually, any of the dimensions are fair game for such backsolving.

Consider what actually happens during backsolving. TK repeatedly iterates on a guessed value, locating the bounding cells in the summary table and then interpolating within the charts, until the result matches the desired solution. In either case, forward or backward, the solution appears almost instantly.

Finally, the model has been set up with units and conversions, allowing users to work in English, Metric, or mixed units. For example, the dimensions might be given in mm but the stress can still be displayed in psi. This units feature also works when backsolving.

Summary

The use of TK Solver greatly simplified the modeling of this relatively complex family of charts. TK's interpolation functions and backsolving capability provide unique tools, allowing solution of problems which might not otherwise be attempted. TK's ability to manage units makes it even more valuable. And of course, TK provides plotting abilities to allow users to study how changes in variables affect other variables.