Case Study: Division By Zero

A customer sent in the following problem involving a five-member truss and reported that he was getting an unexpected error message - “Division by zero” - during iterative solution.

St	Rule
	; reaction equations
	8 * Cy - (PB + PD) * 4 = 0
	-8 * Ay + 4 * (PB + PD) = 0
	Ay - PB - PD + Cy = Erry
	; component lengths of members
	rABx = gBx - gAx
	rABy = gBy - gAy
	rADx = gDx - gAx
	rADy = gDy - gAy
	rBCx = gCx - gBx
	rBCy = gCy - gBy
	rBDx = gDx - gBx
	rBDy = gDy - gBy
	rCDx = gDx - gCx
	rCDy = gDy - gCy
	; lengths of members
	rAB = sqrt(rABx^2 + rABy^2)
	rAD = sqrt(rADx^2 + rADy^2)
	rBC = sqrt(rBCx^2 + rBCy^2)
	rBD = sqrt(rBDx^2 + rBDy^2)
	rCD = sqrt(rCDx^2 + rCDy^2)
	; components of forces
	FABx = FAB * rABx / rAB
	FABy = FAB * rABy / rAB
*	FADx = FAD * rADx / rAD
*	FADy = FAD * rADy / rAD
*	FBCx = FBC * rBCx / rBC
*	FBCy = FBC * rBCy / rBC
*	FBDx = FBD * rBDx / rBD
*	FBDy = FBD * rBDy / rBD
*	FCDx = FCD * rCDx / rCD
>	FCDy = FCD * rCDy / rCD
	FADx + FABx = 0 ; ΣFAx = 0
*	FADy + FABy + Ay = 0 ; ΣFAy = 0
*	-FABx + FBDx + FBCx = 0 ; ΣFBx = 0
*	-FABy - FBDy - FBCy - PB = 0 ; ΣFBy = 0
*	-FCDx - FBCx = 0 ; ΣFCx = 0

The inputs and initial guesses are summarized in the following variable sheet.

St	Input	Name	Output	Unit	Comment
					Applied Loads
	2	PB			
	1	PD			
					Geometric coordinates
	0	gAx			
	0	gAy			
	4	gBx			
	4	gBy			
	8	gCx			
	0	gCy			
	4	gDx			
	0	gDy			
					Member Forces
		FAB	1.41		
		FAD			
		FBC			
		FBD			
>		FCD			
					Reactions
		Ay	1.50		
		Cy	1.50		
		Erry	0.00		Error check on reactions, should be zero
					Lengths of members
		rAB	5.66		
		rAD	4.00		
		rBD	4.00		
		rBC	5.66		
>		rCD	4.00		
					Component lengths of members
		rABx	4.00		
		rABy	4.00		
		rADx	4.00		
		rADy	0.00		
		rBCx	4.00		
		rBCy	-4.00		
		rCDx	-4.00		
>		rCDy	0.00		
		rBDx	0.00		
		rBDy	-4.00		
Gu	1.00	FABx			
		FABy	1.00		
		FADx	-1.00		
		FADy			
		FBCx			
		FBCy			
		FBDx			
		FBDy			
		FCDx			
Gu	1.00	FCDy			

Why the error and how to fix things?

The equation that triggers the error is

FCDy = FCD * rCDy / rCD

We use the Examine Command to determine the current values for each of the variables.

FCDy = 1.000101685964538
FCD = ?
rCDy = 0
rCD = 4

TK sees 1 equation with 1 unknown and tries to solve it. The first step in that process is to multiply both sides of the equation by rCD (4). This results in the following equation.

4.00040674385815 = FCD * 0

It's easy to see that this equation cannot be solved. When TK tries to isolate FCD, it divides 4/0 and its response is the error message, "Division by zero."

The solution to the problem is to choose different guess variables. Let's take a step back and review that process.

If no guesses are supplied the following equations are left unsolved.

St	Rule
	; components of forces
*	FABx = FAB * rABx / rAB
*	FABy = FAB * rABy / rAB
*	FADx = FAD * rADx / rAD
*	FADy = FAD * rADy / rAD
*	FBCx = FBC * rBCx / rBC
*	FBCy = FBC * rBCy / rBC
*	FBDx = FBD * rBDx / rBD
*	FBDy = FBD * rBDy / rBD
*	FCDx = FCD * rCDx / rCD
*	FCDy = FCD * rCDy / rCD
*	FADx + FABx = 0 ; ΣFAx = 0
*	FADy + FABy + Ay = 0 ; ΣFAy = 0
*	-FABx + FBDx + FBCx = 0 ; ΣFBx = 0
*	-FABy - FBDy - FBCy - PB = 0 ; ΣFBy = 0
*	-FCDx - FBCx = 0 ; ΣFCx = 0

The remaining unknowns are all of the variables beginning with F. It is easy to see that all of the equations have at least two unknowns. The variables in these equations having values of 0 are rADy, rCDy, and rBDx. We want to avoid having TK iterate on equations involving these variables.

A single guess for variable FAB of 1 results in a partial solution.

St	Input	Name	Output	Unit	Comment
		FAB	-2.12		
		FAD	1.50		
		FBC			
		FBD			
		FCD			
		FABx	-1.50		
		FABy	-1.50		
		FADx	1.50		
		FADy	0.00		
		FBCx			
		FBCy			
		FBDx			
		FBDy			
		FCDx			
		FCDy			

The unsolved equations are independent of the value of FAB.

St	Rule
*	FBCx = FBC * rBCx / rBC
*	FBCy = FBC * rBCy / rBC
*	FBDx = FBD * rBDx / rBD
*	FBDy = FBD * rBDy / rBD
*	FCDx = FCD * rCDx / rCD
*	FCDy = FCD * rCDy / rCD
*	-FABx + FBDx + FBCx = 0 ; ΣFBx = 0
*	-FABy - FBDy - FBCy - PB = 0 ; ΣFBy = 0
*	-FCDx - FBCx = 0 ; ΣFCx = 0

A second guess variable is selected from the unsolved group. What if we choose FBC? The first equation solves for FBCx. The second equation solves for FBCy. From there, the next equations that can be solved are the three at the bottom because all the others have two unknowns. FBDx, FBDy, and FCDx are computed from the last three equations. At this point, a second pass is made to try to solve the remaining equations.

St	Rule
*	FBDx = FBD * rBDx / rBD
*	FBDy = FBD * rBDy / rBD
*	FCDx = FCD * rCDx / rCD
*	FCDy = FCD * rCDy / rCD

TK tries to solve the first equation in this group for FBD and encounters a division by zero when dividing by rBDx. FBC is not a good guess candidate. Let's try FBD as the second guess variable. The first two equations are now skipped and the 3rd and 4th are solved for FBDx and FBDy. Next, TK solves for FBCx, FBCy, and FCDx in the last three equations and a second pass is made to try to solve the remaining equations.

St	Rule
*	FBCx = FBC * rBCx / rBC
*	FBCy = FBC * rBCy / rBC
*	FCDx = FCD * rCDx / rCD
*	FCDy = FCD * rCDy / rCD

These can all be solved in sequence and none of them have any division by zero problems because none of the r variables equal 0.

We conclude that the second guess variable must be from an equation with one of the zero factors to force the iteration to occur elsewhere. Good guess partners with FAB are then FBD or FCD.

It is interesting to note that if we simply guess all five of the member forces - FAB, FAD, FBC, FBD, and FCD – the solution is found without error. Let's look at the list of unsolved equations again to see why this happens.

St	Rule
	; components of forces
*	FABx = FAB * rABx / rAB
*	FABy = FAB * rABy / rAB
*	FADx = FAD * rADx / rAD
*	FADy = FAD * rADy / rAD
*	FBCx = FBC * rBCx / rBC
*	FBCy = FBC * rBCy / rBC
*	FBDx = FBD * rBDx / rBD
*	FBDy = FBD * rBDy / rBD
*	FCDx = FCD * rCDx / rCD
*	FCDy = FCD * rCDy / rCD
*	FADx + FABx = 0 ; ΣFAx = 0
*	FADy + FABy + Ay = 0 ; ΣFAy = 0
*	-FABx + FBDx + FBCx = 0 ; ΣFBx = 0
*	-FABy - FBDy - FBCy - PB = 0 ; ΣFBy = 0
*	-FCDx - FBCx = 0 ; ΣFCx = 0

When the five guesses are applied, the top 10 equations are all solved for the corresponding x and y terms. Then each of the last five equations generates an error term because all the variables have known values. TK's iterative solver changes the guess values until all five of those error terms get close enough to zero.

Usually we like to suggest that it is possible and even advantageous to try to use a minimum of guesses but it is not a requirement and if it keeps the solver out of "trouble" to use more, feel free to do so.