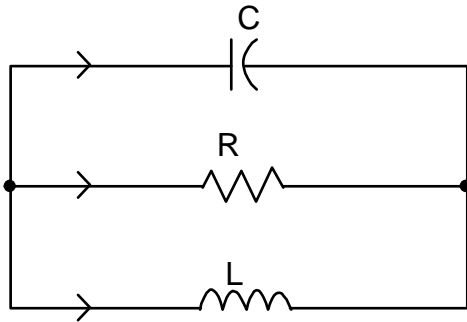


Case Studies: LRC Circuits and Differential Equations

The theory of electric circuits consisting of inductances, resistors, and capacitors, is based on Kirchoff's laws: (1) The net flow of current through each node or junction is zero, (2) the net voltage drop around each loop is zero. In addition to Kirchoff's laws we also have the relation between the current I in amperes through each circuit element and the voltage drop V in volts across the element. Kirchoff's laws and the current-voltage relation for each circuit element provide a system of algebraic and differential equations from which the voltage and current throughout the circuit can be determined as a function of time.

This paper shows how TK Solver can be used to solve such problems numerically. The paper assumes that the reader has already read the available papers on solving differential equations with TK Solver. This is meant as an extension of those papers for this particular subject area.

Case 1: A Simple Parallel LRC Circuit



The differential equations defining this circuit are entered in a TK procedure function named `deq1` with input variables y' , y , and t .

Statement
$y'[1] = -y[2]/C - y[1]/(R*C)$
$y'[2] = y[1]/L$

Here, $y[1]$ and $y[2]$ represent the voltage drop across the capacitor and the current through the inductance. The voltage values will appear in column one of the solution matrix and the current will appear in column two. The expressions $y'[1]$ and $y'[2]$ represent the derivatives of the voltage and current with respect to time. R is the resistance in ohms, C is capacitance in farads, and L is the inductance in henrys. Those three variables are mapped into the procedure function as parameter variables.

The solution matrix list is called S and contains the lists 'V and 'I. A solution table is constructed, including the list t along with V and I. The initial conditions can be input into the table.

Element	t	V	I
1	0	9	.5

A rule is used to call an integrator function. The `ODE_STIFFR` function requires a minimum of five inputs representing the differential equations, the integration limits, the solution matrix, and the independent variable list.

Rule
<code>call ODE_STIFFR('deq1,0,tmax,'S,'t)</code>

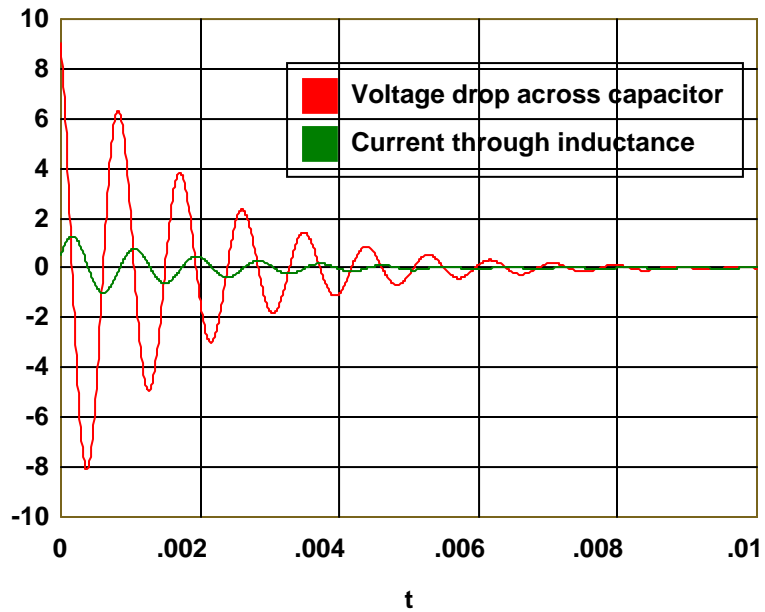
The required inputs are made on the variable sheet. Units are added and conversion factors merged in from the TK Library.

St	Input	Name	Output	Unit	Comment
	45	R		Ohm	Resistance
	.00002	C		F	Capacitance
	.001	L		Hen	Inductance
	.01	tmax		s	Maximum time

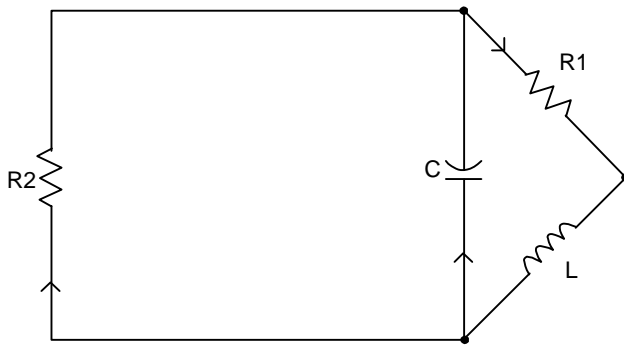
When the model is solved, TK fills the solution table with 856 rows of values. Here are the last few rows.

Element	t	V	I
848	.009818335	.036551838	.002670731
849	.009841381	.032126459	.003463681
850	.009866512	.02647573	.004201643
851	.009896138	.018973883	.004876762
852	.009923247	.011598286	.005291919
853	.009947717	.004761458	.005492197
854	.00997376	-.00246262	.005521753
855	.009997439	-.00878696	.005387917
856	.01	-.00944958	.00536456

Here is the TK plot of V and I vs. t.



Case 2: Solution Independent of Multiple Resistors



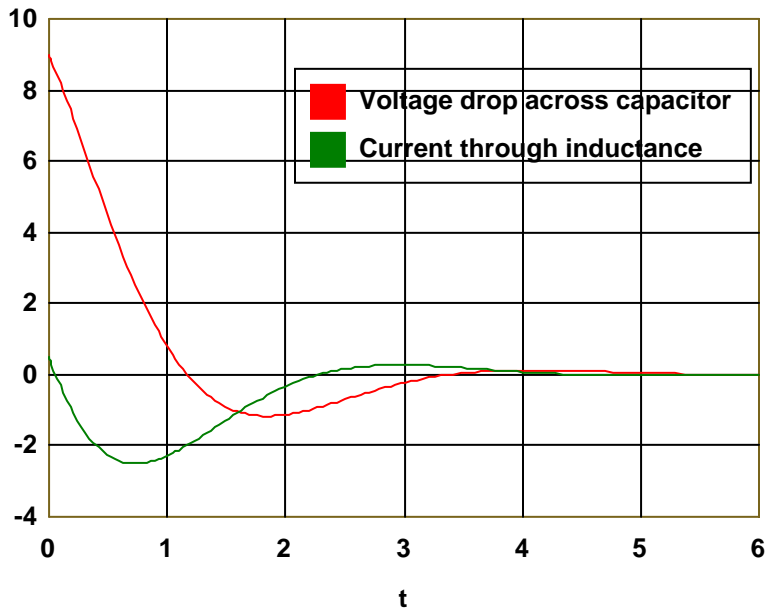
The differential equations defining the current through the inductance and the voltage across the capacitor are as follows...

Statement
$y'[1] = 2*y[2] - y[1]$
$y'[2] = -y[2] - y[1]$

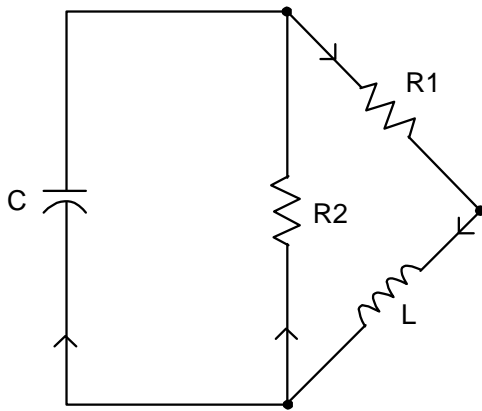
The resistor values have no impact on the two unknown functions.

The following inputs on the Variable Sheet produce the solution shown in the plot below.

St	Input	Name	Output	Unit	Comment
	.00002	C		F	Capacitance
	.001	L		Hen	Inductance
	6	tmax		s	Maximum time



Case 3: Solution Dependent Upon Multiple Resistors



Here are the differential equations defining the voltage through the capacitor and current through the inductance for this circuit.

Statement
$y'[1] = (y[2] - y[1]/R2)/C$
$y'[2] = (-R1*y[2] - y[1])/L$

In this case, the solution is dependent upon the two resistors, so those values are passed into the function as parameter variables. Here is a sample set of inputs and the resulting plot of the solution.

St	Input	Name	Output	Unit	Comment
	45	R1		Ohm	Resistor 1
	30	R2		Ohm	Resistor 2
	.00002	C		F	Capacitance
	.01	L		Hen	Inductance
	.003	tmax		s	Maximum time
	.5	V0		Volt	Initial voltage at t = 0
	.25	I0		Amp	Initial amperage at t = 0

