Case Study -- Variable Pitch Cylindrical Compression Spring

A spring maker presented the following design problem.

Wire Material: Music Wire
Wire diameter: 0.098”
Total Coils: 4.5 (REF)
Coil OD: 0.703”
Free Length: 0.996”
Ends: Closed and Ground
Load when deflected to 0.906”: 13.5 lbf
Load when deflected to 0.759”: 48.75 lbf

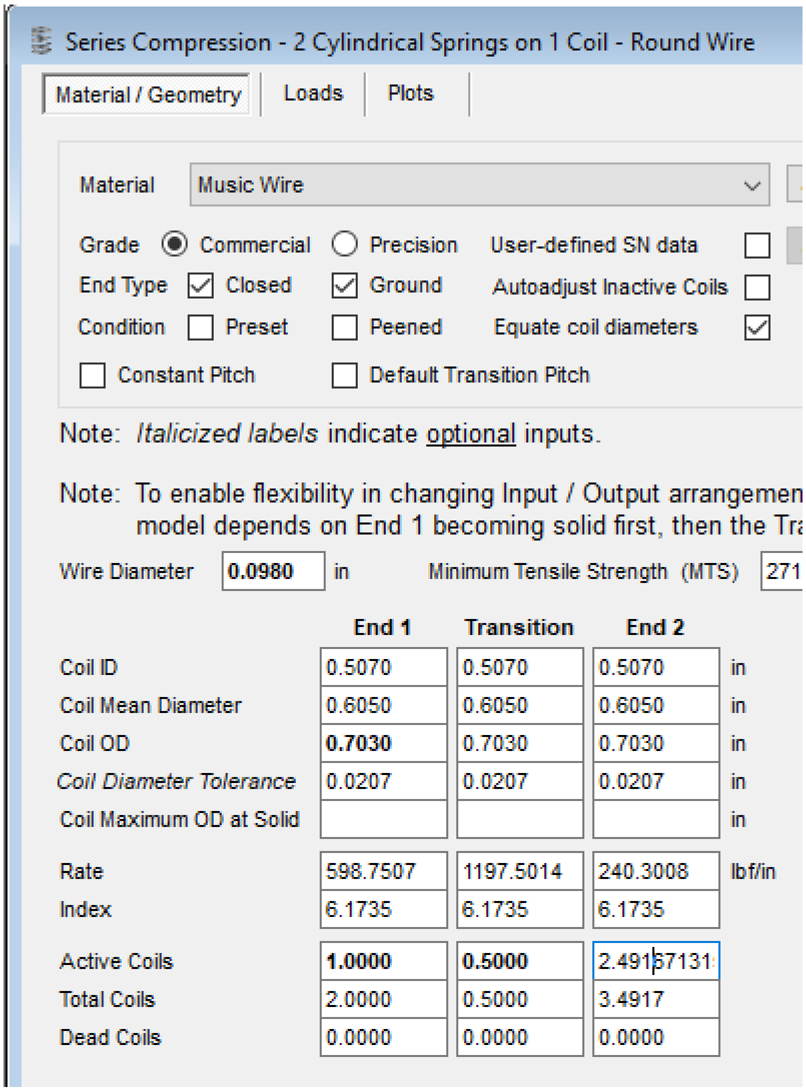
As a quick check, we use the standard round wire compression spring program to see if the smaller deflection condition can be met if all the coils are active.

Wire Diameter	**0.0980**	in	Arbor Diameter	0.4612	in	Active Coils	3.992	
Wire Dia. Tol. (+/-)	0.0010	in	Coil ID	0.5070	in	Total Coils	5.991	
Wire Length	11.4335	in	Coil Mean Diameter	0.6050	in	*Dead Coils*	0.0000	
Wire Weight	0.02449	lb	Coil OD	**0.7030**	in	Pitch	0.2004	in
Minimum Tensile Strength (MTS)	271820	psi	*Coil Dia. Tol. (+/-)*	0.0120	in	Pitch Angle	6.0200	deg
Spring Rate	150.0000	lbf/in	Min. Coil ID (Free)	0.4950	in	*Free Length Tol. (+/-)*	0.0278	in
Spring Index	6.1735		*Shaft Diameter*		in	*Allowable Solid Length*		in
Natural Frequency	944	Hz	*Hole Diameter*		in			
Wire Available	Yes		Next Smaller Wire	0.0970		Next Larger Wire	0.0990	
Estimated Cycle Life								

	Free	Cycle Load 1	Cycle Load 2	Other Load	At Solid	At Buckle	
Load	0	**13.500**			61.336		lbf
Load Tolerance (+/-)	0	4.3740					lbf
Length	**0.9960**	**0.9060**			0.5871		in
Deflection	0	0.0900			0.4089		in
% of Max. Deflection	0	22.0			100		
Corrected Stress	0	27503			124957		psi
Corrected % of MTS	0	10.1			46.0		
Uncorrected Stress	0	22098			100400		psi
Uncorrected % of MTS	0	8.1296			36.9361		
Expanded Coil OD		0.7159			0.7188		in

The spring rate (150 lbf/in) is determined from the inputs for the Free Length and Cycle Load1 values. Then the active coils can be computed. The result, 3.992, is significantly different from the reference value provided (2.5). We will instead assume that approximately 4 coils are initially active until those with a smaller pitch go solid.

We load the program for two springs in series and enter the geometry inputs along with the initial rate and the first load/length combination. We know very little about the distribution of coils and pitches but we do know some things. The program assumes that end1 goes solid before end2 so the pitch is smaller for end1. The 2nd load is considerably greater than the 1st load so we might also assume that there are more coils with the larger pitch. Since there are about 4 active coils initially, we could start with an assumption of 1.5 coils with one pitch and 2.5 coils with a larger pitch. Keep in mind that there are usually an unlimited number of possible configurations given two desired load points. So here are the initial inputs on the Geometry tab (shown in bold).



And here are inputs on the Loads tab.

Series Compression - 2 Cylindrical Springs on 1 Coil - Round Wire

Material / Geometry | **Loads** | Plots

Overall Performance

Active Coils	3.9917	
Initial Rate	150.0000	lbf/in
Wire Length		in
Wire Weight		lb
Free Length	0.9960	in
Free Length Tolerance	0.0445	in

Overall Solid

Load		lbf
Load Tolerance		lbf

Rate Transitions

	Load		Length		Deflection		New Rate	
At 1st Change in Rate		lbf		in		in	200.1392	lbf/in
At 2nd Change in Rate		lbf		in		in	240.3008	lbf/in

Load Cycle - Minimum Load Point 1

Load 1	13.5000	lbf	*Load Tolerance*		lbf
Length	0.9060	in	Deflection	0.0900	in

Load Cycle - Maximum Load Poin

Load 2		lbf	*Load Tol*
Length	0.7590	in	Deflectio

Now if we try different pitch combinations we can see how the load at length 2 changes. If we input 0.15" for the end1 pitch, the resulting load at length 2 is 37 lbf. The program defaults to assume that the pitch of the transition coils will be the average of the coils at each end but this can be overwritten.

	End 1	Transition	End 2	
Pitch	0.1500	0.1867	0.2234	in
Pitch Angle	4.5124	5.6102	6.7038	deg
Free Length	0.2480	0.0934	0.6546	in
Solid Length	0.1960	0.0490	0.3422	in
Solid Load	31.1350	53.1103	75.0857	lbf
Deflection to Solid	0.0520	0.0444	0.3125	in
Stress at Solid	63430	108199	152968	psi
Stress % at Solid	23.3352	39.8053	56.2755	
Wahl Stress Correction Factor	1.2446	1.2446	1.2446	

Load Cycle - Maximum Load Point 2

Load 2	37.0258	lbf	*Load Toleran*
Length	0.7590	in	Deflection

If we reduce the end1 pitch from 0.15 to 0.14, the load 2 value increases to 39 lbf. If we also reduce the pitch of the transition coils to match the pitch of the end1 coils, the load 2 value increases to 41.8. We're heading the right direction.

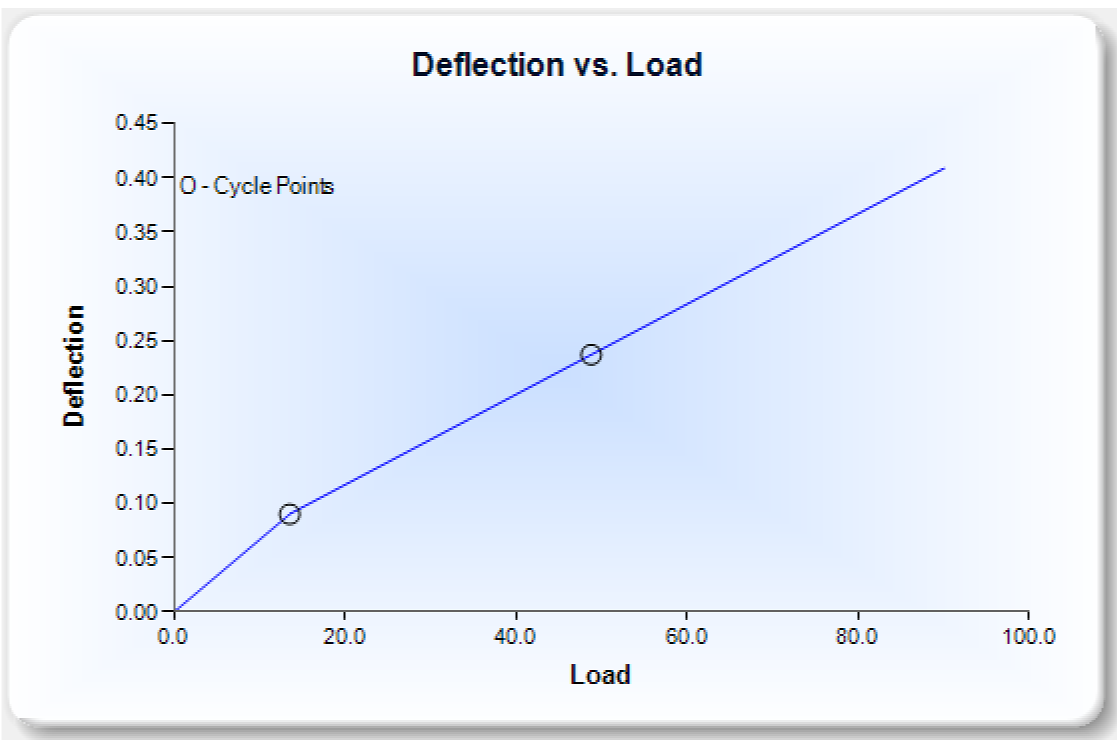
	End 1	Transition	End 2	
Pitch	**0.1400**	**0.1400**	0.2368	in
Pitch Angle	4.2127	4.2127	7.1014	deg
Free Length	0.2380	0.0700	0.6880	in
Solid Length	0.1960	0.0490	0.3422	in
Solid Load	25.1475	25.1475	83.0999	lbf

Load Cycle - Maximum l

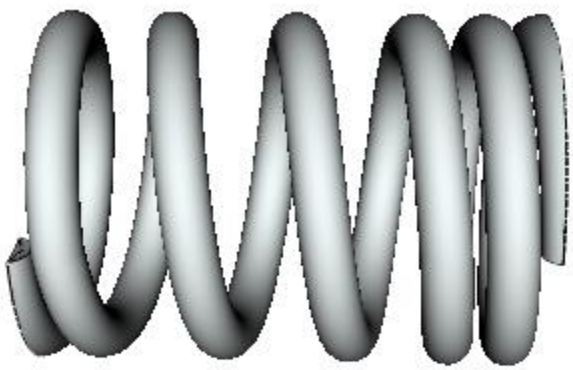
Load 2	41.8123	lbf
Length	**0.7590**	in

Let’s try taking those pitches down to 0.13. That gets load 2 up to 45.4. If we try going to 0.12 we get an error message, Inconsistent. That is because there is no longer enough available pitch for the first load/length combination to work. So we try a pitch of 0.125. That gets load 2 up to 47.2 lbf. We’re almost there.

If we take those pitches down to 0.121, the load 2 value gets to 48.66. A few more tweaks and with a pitch at one end of 0.1207, the load 2 value is 48.77. The pitch at the other end is 0.2484. Here is a plot of the deflection vs. load.



Here is what our spring looks like.



As mentioned, this is not a unique solution. We specified. 1.5 coils at the smaller pitch and roughly 2.5 coils at the larger pitch. If we increase the number of end 1 coils from 1.5 to 1.7, we find that the end 1 pitch increases from 0.1207 to 0.1276 to get load 2 to line up.

And here's another combination that works.

	End 1	Transition	End 2	
Coil ID	0.5070	0.5070	0.5070	in
Coil Mean Diameter	0.6050	0.6050	0.6050	in
Coil OD	**0.7030**	0.7030	0.7030	in
Coil Diameter Tolerance	0.0207	0.0207	0.0207	in
Coil Maximum OD at Solid	0.7246	0.7246	0.7301	in
Rate	299.3753	1197.5014	401.3959	lbf/in
Index	6.1735	6.1735	6.1735	
Active Coils	**2.0000**	**0.5000**	1.4917	
Total Coils	3.0000	0.5000	2.4917	
Dead Coils	0.0000	0.0000	0.0000	

	End 1	Transition	End 2	
Pitch	**0.1442**	**0.1442**	0.2946	in
Pitch Angle	4.3386	4.3386	8.8117	deg
Free Length	0.3864	0.0721	0.5375	in
Solid Length	0.2940	0.0490	0.2442	in
Solid Load	27.6623	27.6623	117.7359	lbf
Deflection to Solid	0.0924	0.0231	0.2933	in
Stress at Solid	56355	56355	239857	psi
Stress % at Solid	20.7324	20.7324	88.2411	
Wahl Stress Correction Factor	1.2446	1.2446	1.2446	

Also note however, that as we reduce the number of end 2 coils, the stress on those coils increases. Our initial solution is probably better because the stresses are better balanced.