Optimization Case Study – Pipeline Transmission System

This example is taken from Engineering Optimization, Methods and Applications by G.V. Reklaitis, A Ravindran, and K. M. Ragsdell, published in 1983 by John Wiley & Sons.

A natural gas pipeline transmission system is required to pump 100 MMSCF/day of natural gas through a distance of 600 miles. Compressor stations are to be placed at equal distances. The design variables are the pipe diameter $D$, compressor discharge and inlet pressures $P_1$ and $P_2$, and length between stations $L$. The optimum design should be such that the total cost of the pipeline facility is minimized.

Here are the equations.

$$R = 1.987$$

$$r = \frac{P_1}{P_2}$$

$$X = r^{0.219} - 1$$

$$\text{hp} = 27168 \cdot X$$

$$C_{\text{fixed}} = \frac{4.5E6}{L} + \frac{7.08E8}{L} \cdot X$$

$$C_{\text{fuel}} = \frac{5.82E8}{L} \cdot X$$

$$C_{\text{main}} = \frac{0.58E8}{L} \cdot X$$

$$C_{\text{labor}} = \frac{8.64E6}{L} + \frac{1.96E8}{L} \cdot X$$

$$P_{\text{install}} = 73800 \cdot D$$

$$t = \frac{D \cdot P_1}{50000}$$

$$P_{\text{matl}} = 15.68 \cdot D^2 \cdot P_1$$

$$T = P_{\text{matl}} + 9E5 + P_{\text{install}} + C_{\text{fixed}} + C_{\text{fuel}} + C_{\text{main}} + C_{\text{labor}}$$

$$f = 0.008 \cdot D^{-\frac{1}{3}}$$

$$Q = 3.39 \cdot \sqrt{\frac{P_1^2 - P_2^2}{f} \cdot \frac{5}{L}}$$

Here is a summary of the variables, with descriptions and solution values.
The TK Solver Optimizer is concisely configured as follows.

### Optimization Parameters

- **Set Target Variable:** 
- **Equal To:** 
- **By Changing Variables:** L, P1, D
- **Subject to the Constraints:** 
  1. L ≥ 1
  2. P1 ≥ 1
  3. r ≥ 1.01

Here is a TK table containing the optimal solutions for a range of flow rates.

<table>
<thead>
<tr>
<th>Element</th>
<th>Q</th>
<th>L</th>
<th>D</th>
<th>P1</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10,000,000</td>
<td>35.6628</td>
<td>37.0435</td>
<td>190.584</td>
<td>9,784,880</td>
</tr>
<tr>
<td>2</td>
<td>9,000,000</td>
<td>37.4409</td>
<td>35.2846</td>
<td>200.092</td>
<td>9,363,160</td>
</tr>
<tr>
<td>3</td>
<td>8,000,000</td>
<td>39.5325</td>
<td>33.4177</td>
<td>211.269</td>
<td>8,915,380</td>
</tr>
<tr>
<td>4</td>
<td>7,000,000</td>
<td>42.0454</td>
<td>31.4203</td>
<td>224.7</td>
<td>8,436,300</td>
</tr>
<tr>
<td>5</td>
<td>6,000,000</td>
<td>45.1458</td>
<td>29.2652</td>
<td>241.275</td>
<td>7,918,750</td>
</tr>
<tr>
<td>6</td>
<td>5,000,000</td>
<td>49.1369</td>
<td>26.8952</td>
<td>262.609</td>
<td>7,352,300</td>
</tr>
<tr>
<td>7</td>
<td>4,000,000</td>
<td>54.4442</td>
<td>24.2651</td>
<td>291.021</td>
<td>6,720,860</td>
</tr>
<tr>
<td>8</td>
<td>3,000,000</td>
<td>62.1899</td>
<td>21.2467</td>
<td>332.413</td>
<td>5,997,170</td>
</tr>
<tr>
<td>9</td>
<td>2,000,000</td>
<td>74.9624</td>
<td>17.6247</td>
<td>400.602</td>
<td>5,127,170</td>
</tr>
<tr>
<td>10</td>
<td>1,000,000</td>
<td>103.22</td>
<td>12.799</td>
<td>551.595</td>
<td>3,969,820</td>
</tr>
</tbody>
</table>
The table was created in a few simple steps. First, each of the variables in the table was assigned the List status on the variable sheet. The flow rate was input and the Optimizer run. The results were then placed into lists using TK’s Put in Lists Command. These steps were repeated for each of the desired flow rates.

The lists can also be plotted, as shown below.